

Reliability analysis of slender reinforced concrete column using probabilistic SBRA method

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ABSTRACT. Modern supporting reinforced concrete structures, i.e. columns, piers, frame legs, etc., are characteristics by their higher slenderness. Effects of external forces on these structures are significantly dependent on their deformations. In the reliability assessment of these structures it is necessary to deal with the equilibrium of internal and external forces acting on deformed structure. Rapidly growing potential of computer technology allows utilize fully probabilistic methods of reliability assessment of structures. The subject of this paper is probabilistic safety assessment of slender reinforced concrete column loaded by eccentric compressive force. The solution is based on fully probabilistic SBRA method (see e.g. [1], [2]) using Monte Carlo simulation technique.

Keywords: safety assessment, reinforced concrete, slender column

1. Introduction

Slender, eccentrically compressed members are in modern reinforced concrete structures still often used. These members can be either stand-alone supports (columns, piers) or they can be parts of structural systems. The effect of external forces on these structures is non-negligibly influenced by its deformation. Due to this, the reliability assessment is more complicated, especially in case of higher slenderness. Knowledge acquired from the research of deformation characteristics of concrete and steel during various ways of loading and available powerful computers allows apply fully probabilistic methods of reliability assessment of these slender structures. Application of probabilistic reliability methods allows analyze more complex problems with the direct determination of probability of failure P_f . On the following pages is demonstrated safety analysis of slender reinforced concrete column exposed to eccentric compressive force parallel to the longitudinal axis of the column. The solution is based on the fully probabilistic method SBRA (Simulation-Based Reliability Assessment) and direct Monte Carlo simulation technique (see [1], [2]).

2. Equilibrium on a slender eccentrically loaded member

Equilibrium of internal and external forces in the cross-section of bended column can be followed if we express the bending moment M as the function of independently variable deflection y_0 of the free end of the column fixed in the foot. The deflection y_0 is dependent on the member curvature $y'' = 1/r$ at the considered cross-section (r is radius of curvature of bending curve):

$$y_0 := f(y'') \quad (1)$$

The parameter is load F_n (external force N).

If we express approximately deflection y_0 by the linear function of the member curvature y'' at the considered cross-section:

$$y_0 := k \cdot y'' \quad (2)$$

where k is constant dependent on the shape of the member bending curve, the relationship „ $M - y''$ “ can be used as a base for fitting approximate solution (for more information see [5], [3]). Deviations due to the linear approximation (2) are relatively small up to the slenderness ratio about 150. Above this level, the differences among the shapes of deflecting curves corresponding to various member curvatures become significant and the dependency (2) comes to common form (1), so the lines are changed to curves. Their shape is dependent on physical non-linearity of the member along its length, so the shape is dependent not only on magnitude of the force F_n , but also on its initial eccentricity e_0 .

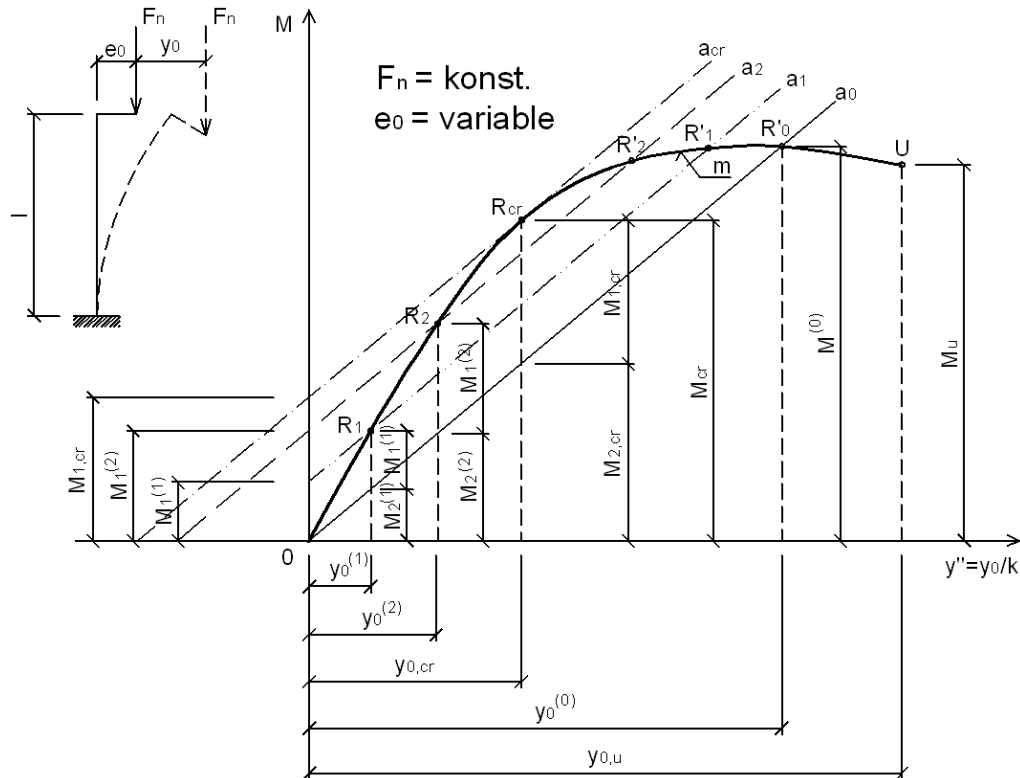


Fig. 1. Relationship $M - y''$ at the foot cross-section of the member from physically non-linear material, where $F_n = \text{const.}$ and $y_0 = ky''$

Relationship between internal and external forces at the foot cross-section of intangible member consisted of physically non-linear material with the consideration of (2) and acting constant force $F_n = N$ is possible express by Fig.1.

The curve “m” expresses the course of bending moments of internal forces M_r with the simultaneous action of centric internal force N on the curvature y'' at the foot cross-section of the member. Lines a_0, a_1, \dots, a_{cr} represent dependency of total bending moments $M = M_1 + M_2$ external forces on the deflection y_0 of the free end of the member. Segments on axis M correspond to initial bending moments $M_1 = F_n e_0$. The equilibrium of internal and external forces is achieved at the points of intersections of curve “m” and lines $a_0, a_1, a_2, \dots, a_{cr}$. Points R_0, R_1, R_2 express their stable equilibrium, whereas points R'_0, R'_1, R'_2 correspond to unstable equilibrium.

As soon as the unstable equilibrium is achieved and during increased deflection the effect of external forces prevails, the deflection increases up to the cross-section failure.

If the eccentricity e_0 and the initial bending moment M_1 increase in such way, that line $a'_1 = a_{cr}$ touches the curve “m” at the point R_{cr} , the equilibrium of internal and external forces is only unstable. This initial eccentricity $e_0 = e_{cr}$ is noted as critical for the force F_n . Curve “m” is finished at the point U , which ordinate corresponds to the moment of internal forces at the exhaustion of carrying capacity of the cross-section (i.e. the highest deformation of the compressed concrete or tensioned reinforcement reaches the limit value).

3. Safety assessment of slender reinforced column by SBRA method

3.1 Assignment

In the following paragraphs there is demonstrated example focused on the reliability (safety) assessment of slender reinforced column fixed in the foot and free in the head. The column of rectangular cross-section is loaded by eccentric compressive force at the top (see Fig.2). Input quantities, including loads, are considered as random variables (see Tab.1). Graphical representation of its probabilistic functions is in Fig.3, 4. and 5.

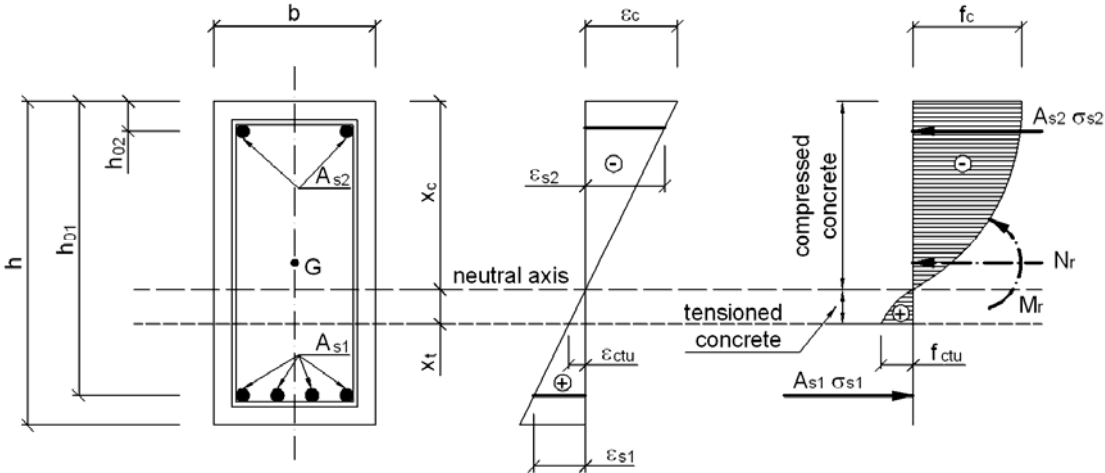


Fig.2. Scheme of reinforced cross-section exposed to eccentric compressive force

Tab. 1. Input quantities

Quantity	Mark	Nominal value	Histogram	Unit
Normal force due to load	N_f	-265	Nf.his	kN
Bending moment due to load	M_f	215	Mf.his	kNm
Cross-section width	b	300	N1-05.his	mm
Cross-section height	h	400	N1-05.his	mm
Column length	l	4000	N1-01.his	mm
Diameter of reinforcing bars „1“	ϕ_1	16	N1-03.his	mm
Diameter of reinforcing bars „2“	ϕ_2	16	N1-03.his	mm
Number of reinforcing bars „1“	n_{s1}	4	-	-
Number of reinforcing bars „2“	n_{s2}	2	-	-
Distance of reinforcing bars „1“ from compressed edge	h_{01}	360	N1-05.his	mm
Distance of reinforcing bars „2“ from compressed edge	h_{02}	40	N1-05.his	mm
Strength of concrete in compression	f_c	1	C30-fc.his	MPa
Strength of concrete in tension	f_{ct}	1	C30-fct.his	MPa
Steel yield	f_y	1	R10505.his	MPa
Limit strain of reinforcement	ϵ_{su}	1	-	%
Modulus of elasticity of the reinforcement	E_s	210	-	GPa

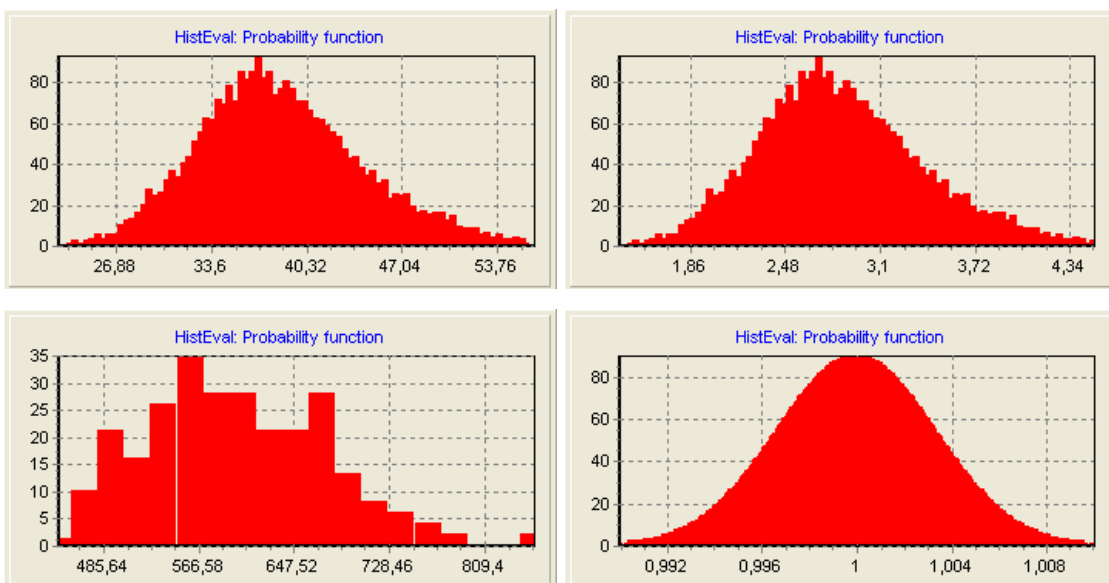


Fig. 3. Bounded histograms C30-fc.his, C30-fct.his, R10505.his and N1-01.his

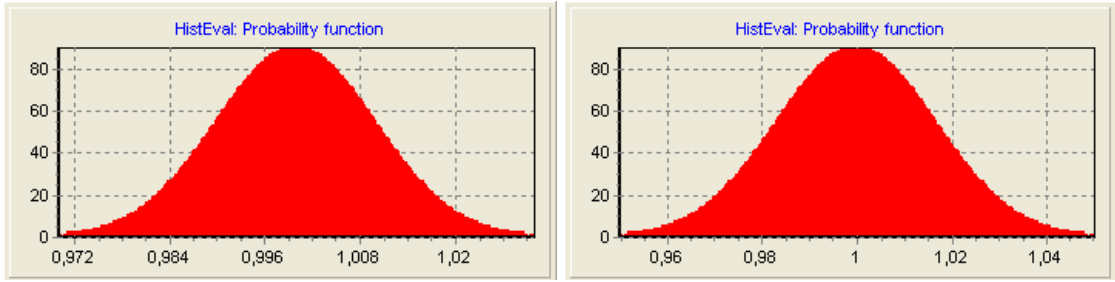


Fig. 4. Bounded histograms *N1-03.his* and *N1-05.his*

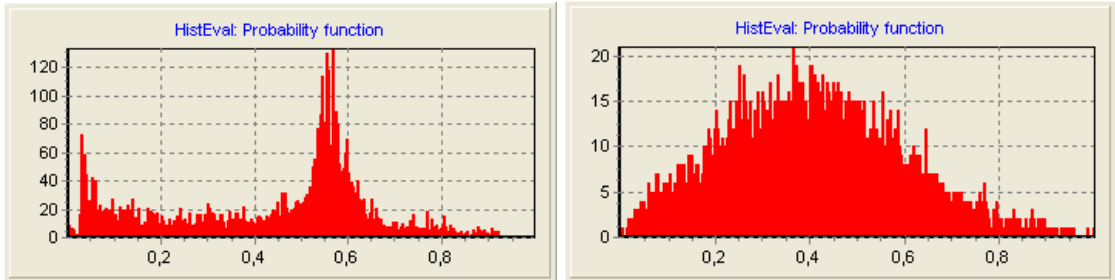


Fig. 5. Bounded histograms *Nf.his* and *Mf.his*

3.2 Calculation the critical bending moment M_{cr}

Critical eccentricity of acting compressive force can be determined using the cross-section diagram $M - y''$ (see chapter 2). Probabilistic calculation of such diagram corresponding to considered input data is demonstrated in the paper [7] published as well in this conference proceeding. Considering approximately second order bending moment by the formula:

$$M2(y'', N_f) := \overrightarrow{(0.1 \cdot |N_f| \cdot l_0(l)^2 \cdot y'')} \quad (3)$$

where the effective length l_0 is

$$l_0(l) := 2 \cdot l \quad (4)$$

we can determine critical curvature y_{cr} corresponding to the critical eccentricity e_{cr} of the compressive force N_r (see Fig.1).

By the substitution of critical curvature y_{cr} into formulas (13) in [7] and (3) we can determine the first order bending moment corresponding to the critical curvature e_{cr} :

$$M1_{cr}(N_f) := M_{cr}(N_f) - M2_{cr}(N_f) \quad (5)$$

3.3 Reliability (safety) assessment of the column

Safety of considered column can be proved using the reliability function:

$$RF_{M1.cr}(N_f) := M1_{cr}(N_f) - M1_f \quad (6)$$

and the criterion reliability:

$$P_f < P_d, \quad (7)$$

where P_f is probability of failure calculated from the reliability function (6) and P_d is design probability, which should be given in corresponding code (see e.g. (viz [6])).

3.4 Results of simulation

Due to random character of input quantities the output ones have certain randomness as well. Calculated probabilistic functions of followed output quantities are shown in Figs. 6-9.

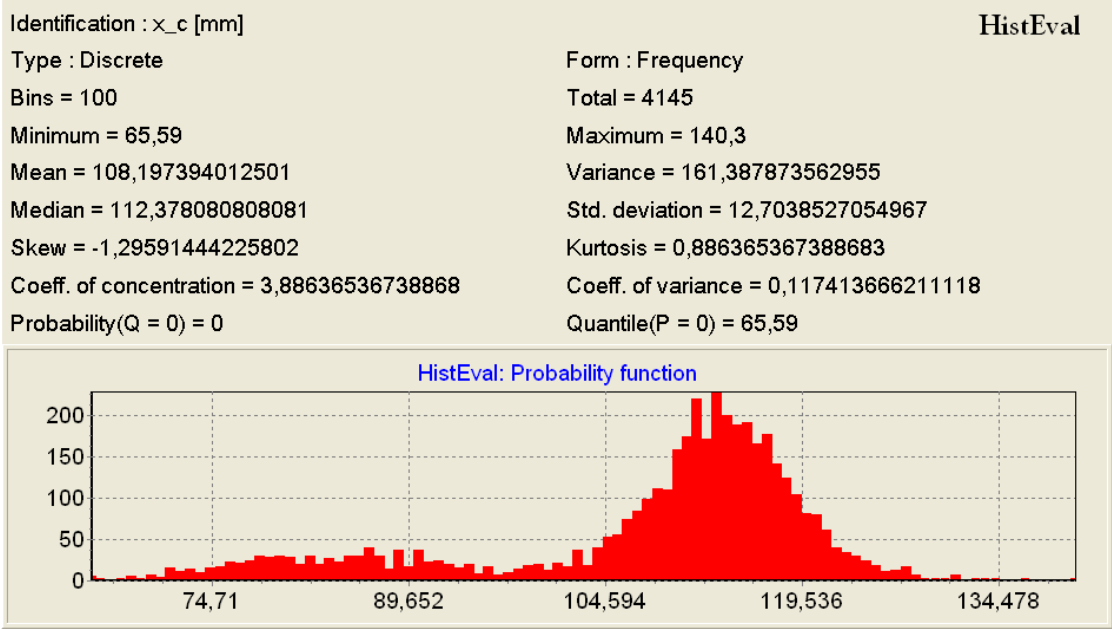


Fig. 6. Distance x_c [mm] of the neutral axis from the compressed edge

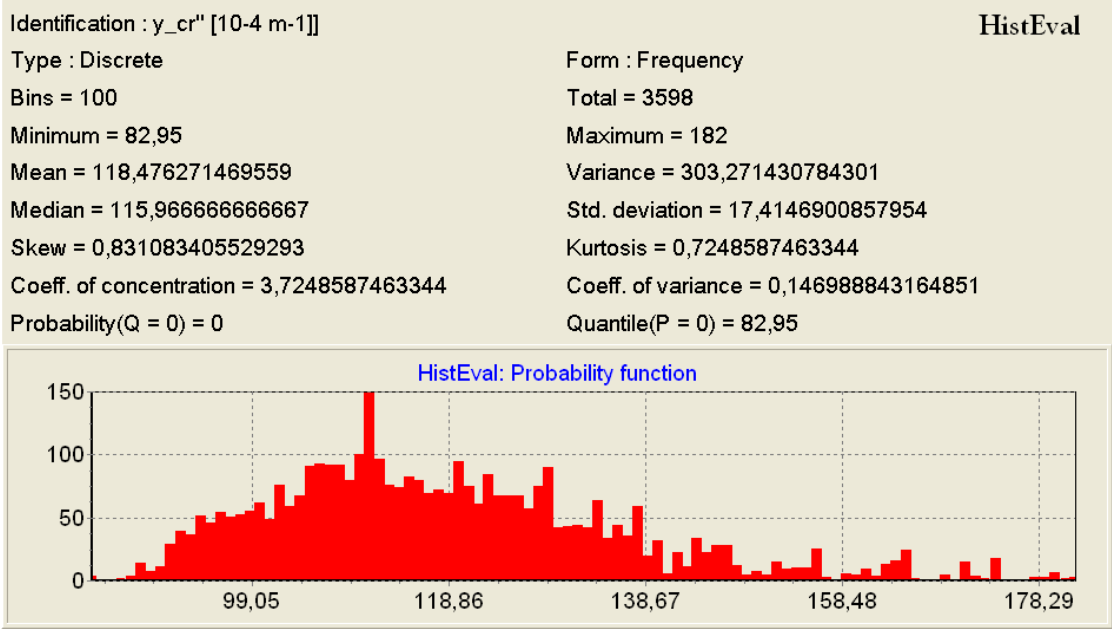


Fig. 7. Critical curvature y_{cr}'' [$10^{-4} m^{-1}$]

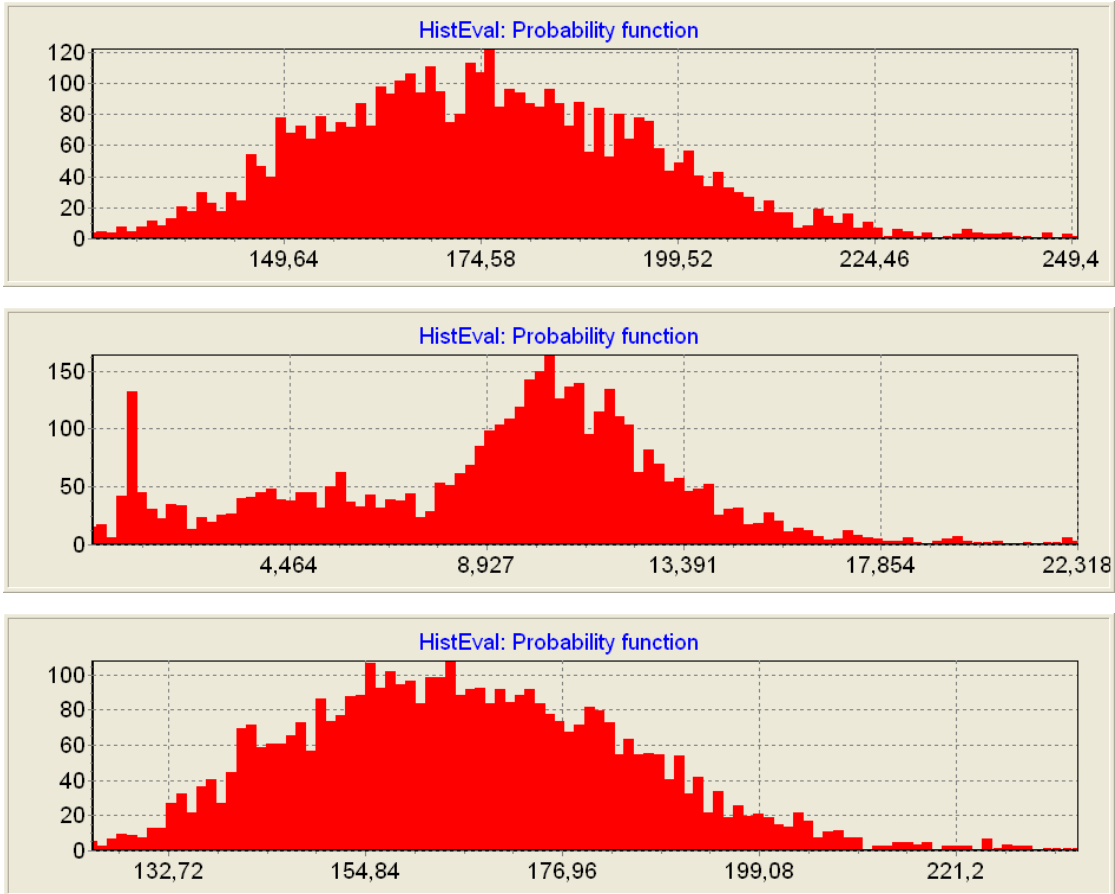


Fig. 8. Histograms of critical moments M_{cr} , $M_{2,cr}$ and $M_{1,cr}$ corresponding to the y_{cr} "

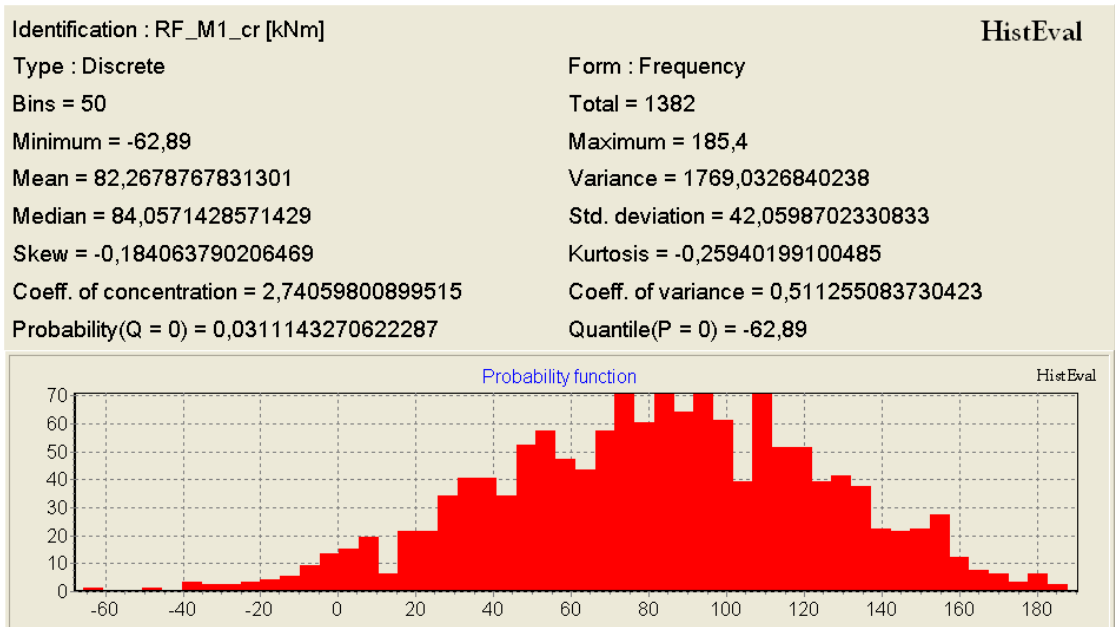


Fig. 9. Safety function corresponding to the 1st order bending moment $M_{1,cr}$

4. Conclusions

The paper demonstrates the possible way of fully probabilistic safety assessment of slender reinforced column loaded by eccentric axial force. Random character of input quantities is respected by using of corresponding bounded histograms, which usually better approximate actual randomness of these quantities. Working stress-strain diagram of compressed and tensioned concrete are described by parabolic curves, the stress-strain diagram of steel is considered as bilinear. The outlined solution allows assess the safety of slender columns with the consideration of second order load effects. The applied probabilistic approach (unlike the partial factors method utilized in most today's codes) leads to direct determination of probability of failure. The reliability can be proved using the corresponding reliability criterion (7).

Results published in this paper and the paper [7] correspond to thousands of simulation steps. At the deadline for sending papers to publisher it was not possible to perform more simulations. For the safety assessment of given example would be necessary millions of simulation steps. The attention of authors will be in the next given to the development of appropriate software allowing rapid simulation achieving millions simulations in short time.

In the solved example there was not also considered creep of the concrete. It is supposed, that the model for calculation of this effect will be incorporated into prepared new software.

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References

1. Marek, P., Guštar, M., Anagnos, T. (1995): *Simulation-Based Reliability Assessment for Structural Engineers*, CRC Press, Inc., Boca Raton, Florida.
2. Marek, P., Brozzetti, J., Guštar, M., Tikalsky, J. P. (2003): *Probabilistic Assessment of Structures using Monte Carlo Simulation. Background, Exercises and Software. Second Edition*. Institut of Theoretical and Applied Mechanics, Academy of Sciences of the Czech Republic, Praha.
3. Janda, L., Křístek, V., Kvasnička, M., Procházka, J. (1983) *Štíhlé betonové tlačené pruty*. Praha: SNTL - Nakladatelství technické literatury.
4. CEB-FIP (1978) *Model Code for Concrete Structures*. Bulletin d'Information No 125-E, Paris.
5. CEB (1977) *Design Manual on Buckling*, Bulletin d'Information No 123, Decembre, Paris.
6. ČSN 731401 (1998): *Navrhování ocelových konstrukcí*. Praha: Český normalizační institut.
7. Pustka, D. Čajka, R., Kubečka, K., Marek, P. (2008). Probabilistic calculation of reinforced concrete cross-section diagram " $M_r(x) - y''(x)$ " using SBRA method. In: *Reliability, safety and diagnostics of transport structures and means 2008*. University of Pardubice, Czech Republic, 25-26 September 2008.
8. Pustka, D. HistEval [computer program]. Ver. 1.0.0 EDU. Technical University of Ostrava: 2008.