Priority Queues and Heaps

Metodický koncept k efektivní podpoře klíčových odborných kompetencí s využitím cizího jazyka ATCZ62 - CLIL jako výuková strategie na vysoké škole

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Priority Queue – ADT

- A priority queue stores a collection of items
- An item is a pair
  (key, element)
- Main methods of the Priority Queue ADT
  - **insertItem(k, o) -** inserts an item with key k and element o
  - **removeMin() -** removes the item with smallest key and returns its element
- Additional methods
  - **minKey(k, o)** **minElement()** **size()** **isEmpty()**
- Applications:
  - Standby flyers, Auctions, Stock market

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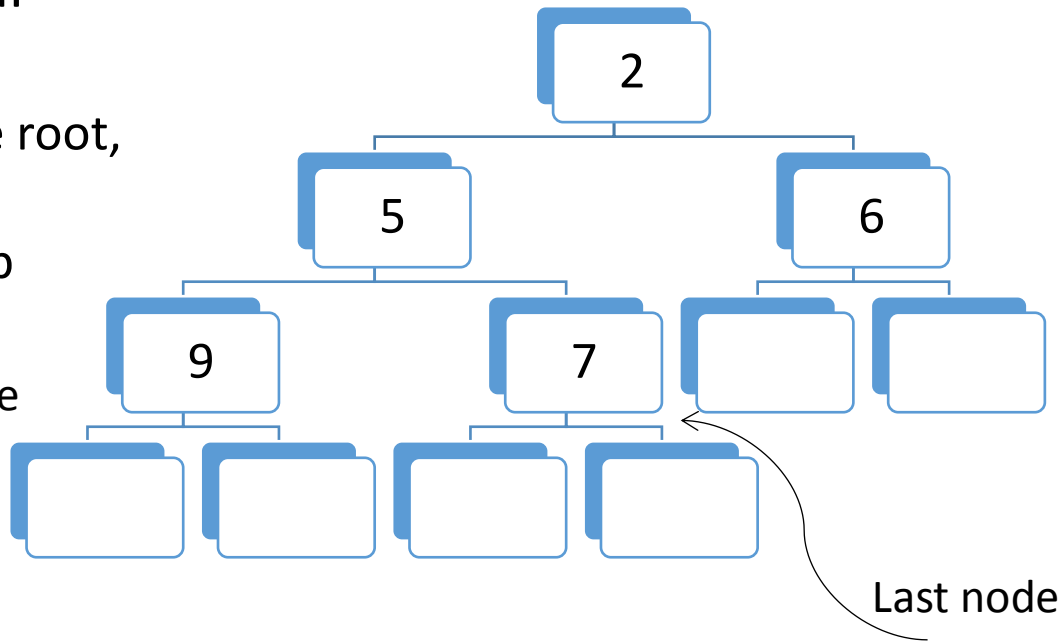
Priority Queue

- Keys in a priority queue can be arbitrary objects on which an order is defined
- Two distinct items in a priority queue can have the same key
- Mathematical concept of total order relation $\leq$
  - Reflexive property: $x \leq x$
  - Antisymmetric property: $x \leq y \wedge y \leq x \Rightarrow x = y$
  - Transitive property: $x \leq y \wedge y \leq z \Rightarrow x \leq z$
- Comparator – ADT
  - A comparator encapsulates the action of comparing two objects according to a given total order relation

Heap

- A heap is a binary tree storing keys at its internal nodes and satisfying the following properties:
  - Heap-Order: for every internal node v other than the root, $key(v) \geq key(parent(v))$
  - Complete Binary Tree: let h be the height of the heap
    - for $i = 0, \ldots, h-1$, there are 2^i nodes of depth i
    - at depth $h-1$, the internal nodes are to the left of the external nodes
- The last node of a heap is the rightmost internal node of depth $h-1$



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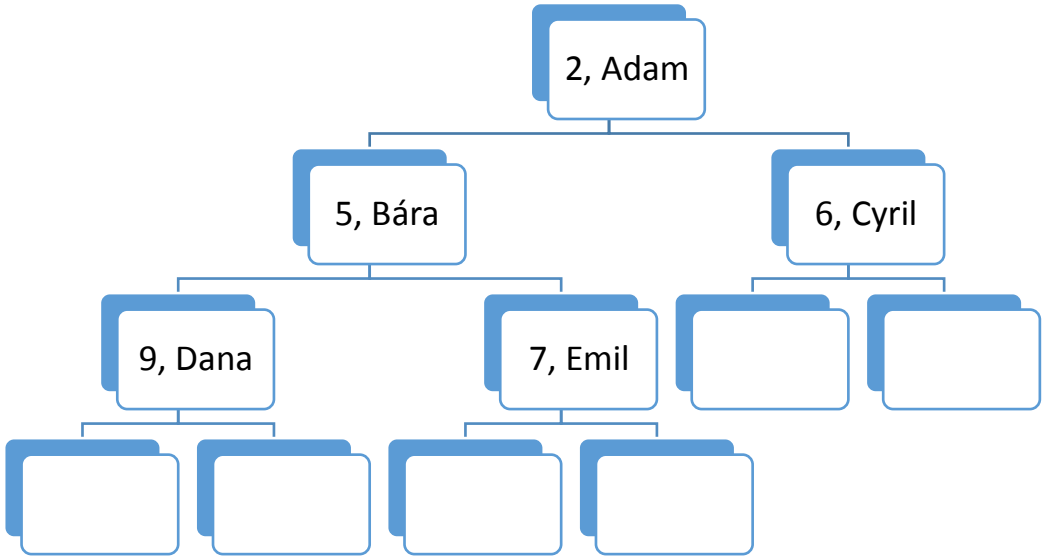
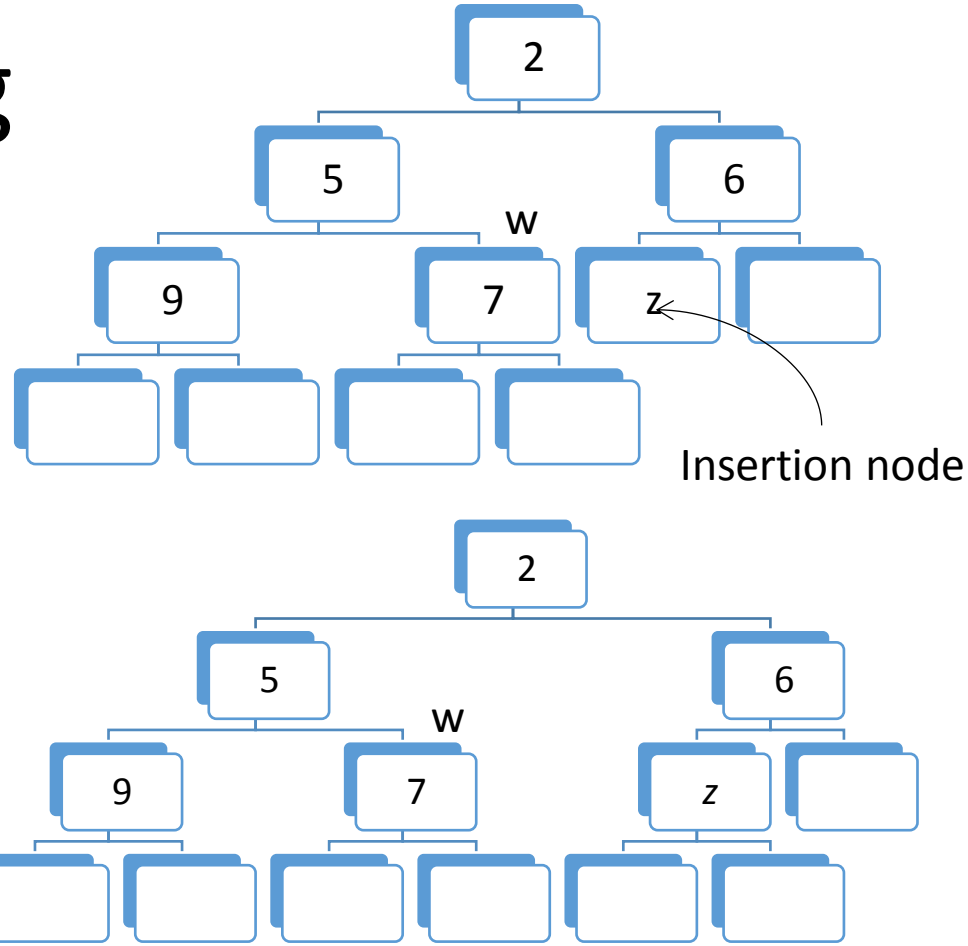
Heaps and Priority Queue

- We can use a heap to implement a priority queue
- We store a (key, element) item at each internal node
- We keep track of the position of the last node



Heap – inserting and deleting

- The insertion algorithm consists of three steps
  - Find the insertion node z (the new last node)
  - Store k at z and expand z into an internal node
  - Restore the heap-order property (discussed next)
- The removal algorithm consists of three steps
  - Replace the root key with the key of the last node w
  - Compress w and its children into a leaf
  - Restore the heap-order property (discussed next)



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Restoring the heap order

- **upheap()**
  - After the insertion of a new key k, the heap-order property may be violated
  - Algorithm upheap restores the heap-order property by swapping k along an upward path from the insertion node
  - Upheap terminates when the key k reaches the root or a node whose parent has a key smaller than or equal to k
- ***downheap()***
  - After replacing the root key with the key k of the last node, the heap-order property may be violated
  - Algorithm downheap restores the heap-order property by swapping key k along a downward path from the root
  - Upheap terminates when key k reaches a leaf or a node whose children have keys greater than or equal to k

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