Graph theory

Metodický koncept k efektivní podpoře klíčových odborných kompetencí s využitím cizího jazyka ATCZ62 - CLIL jako výuková strategie na vysoké škole

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Graph theory

- A graph is a pair (V, E), where
  - V is a set of nodes, called vertices
  - E is a collection of pairs of vertices, called edges
  - Vertices and edges are positions and store elements
- Types of edges
  - Directed – ordered pair of vertices (u,v), first vertex u is the origin, second vertex v is the destination
  - Undirected - unordered pair of vertices (u,v)
  - Loops – edge that connects a vertex to itself
  - Multiple edges – between edges (u,v) is more than one edge

Graph theory

- Types of graphs
  - Directed – all edges are direct
  - Undirected – all edges are undirected
  - Multigraph – contains multiple edges
- Terminology
  - End vertices (or endpoints) of an edge
  - Edges incident on a vertex
  - Adjacent vertices
  - Degree of a vertex
  - Parallel edges
  - Self-loop

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Graph theory

- Path
  - sequence of alternating vertices and edges
  - begins with a vertex
  - ends with a vertex
  - each edge is preceded and followed by its endpoints
- Simple path
  - path such that all its vertices and edges are distinct
- Cycle
  - circular sequence of alternating vertices and edges
  - each edge is preceded and followed by its endpoints
- Simple cycle
  - cycle such that all its vertices and edges are distinct

Graph theory

- Electronic circuits
  - Printed circuit board
  - Integrated circuit
- Transportation networks
  - Highway network
  - Flight network
- Computer networks
  - Local area network
  - Internet
  - Web
- Databases
  - Entity-relationship diagram

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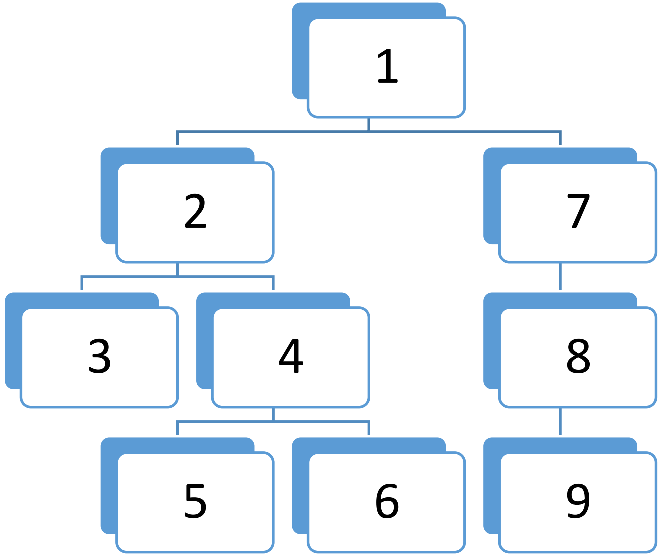
Graph – ADT

- Accessor methods
  - **aVertex()**
  - **incidentEdges(v)**
  - **endVertices(e)**
  - **isDirected(e)**
  - **origin(e)**
  - **destination(e)**
  - **opposite(v,e)**
  - **areAdjecent(v,w)**
- Update methods
  - **insertVertex(o)**
  - **insertEdge(v, w, o)**
  - **insertDirectedEdge(v, w, o)**
  - **removeVertex(v)**
  - **removeEdge(e)**
- Generic methods
  - **numVertices()**
  - **numEdges()**
  - **vertices()**
  - **edges()**

Graph – DFS – depth-first search

- for traversing or searching tree or graph data structures. One starts at the root (selecting some arbitrary node as the root in the case of a graph) and explores as far as possible along each branch before backtracking.

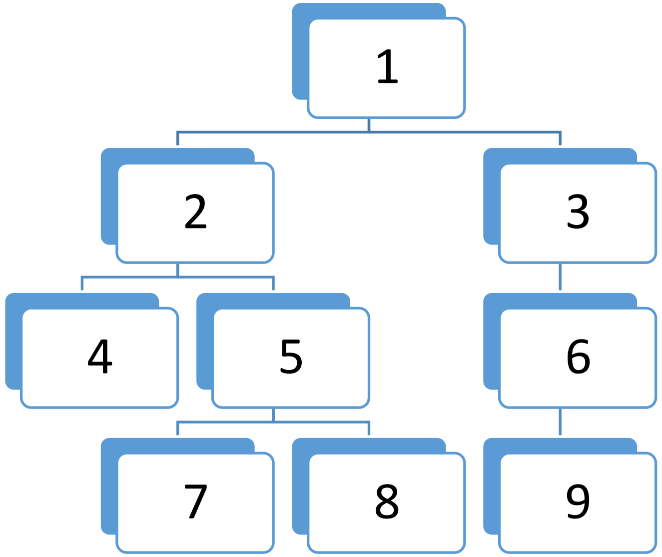
```
procedure DFS-iterative(G,v):
    let S be a stack
    S.push(v)
    while S is not empty
        v = S.pop()
        if v is not labeled as discovered:
            label v as discovered
            for all edges from v to w in G.adjacentEdges(v) do
                S.push(w)}
```



Graph – BFS – Breadth-first search

- traversing or searching tree or graph data structures. It starts at the tree root (or some arbitrary node of a graph, sometimes referred to as a 'search key') and explores the neighbor nodes first, before moving to the next level neighbors.

```
Breadth-First-Search(Graph, root):
  create empty set S
  create empty queue Q
  add root to S
  Q.enqueue(root)
  while Q is not empty:
    current = Q.dequeue()
    if current is the goal:
      return current
    for each node n that is adjacent to current:
      if n is not in S:
        add n to S
        n.parent = current
        Q.enqueue(n)
```



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Graph – shortest path

- Dijkstra's algorithm
  - non-negative weights on the edges
  - $O(|V|^2+|E|)$ – V number of vertices, E number of edges
- Bellman-Ford algorithm
  - Graph can have negative edges
  - $O(V \cdot E)$ – slower than Dijsktra's alg.
- Floyd-Warshall algorithm
  - Directed graph with non-negative edges
  - Find shortest path between all vertices
  - Time complexity – $O(V^3)$, memory complexity - $O(V^2)$

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Graph – shortest path

- Johnson's algorithm
  - find the shortest paths between all pairs of vertices in a sparse, edge weighted, directed graph. It allows some of the edge weights to be negative numbers
  - $O(V^2 \log_2(V) + VE)$